Description Logic in a nutshell

Seminar „Resources for Computational Linguists“
SS 2007

Magdalena Wolska & Michaela Regneri
Motivation

• We have seen all those great ontologies - how can we make use of them?

• How can we add logic inference to our world knowledge? (Aristotle is a human, humans are mortal -> Aristotle is mortal)

• How can we do all that without having to wait for ages?
Outline

• Some curses of FOL

• Some solutions: Description Logics
  • Basics and Terms
  • Reasoning: RACER
Some curses of FOL

• FOL is not decidable

Provide a system with the following:
(The universe shall consist of natural numbers)

\[ \forall x \exists y \text{ bigger}_\text{than}(x,y) \]
\[ \forall x \forall y \forall z ((\text{bigger}_\text{than}(x,y) \land \text{bigger}_\text{than}(y,z)) \rightarrow \text{bigger}_\text{than}(x,z)) \]

Finding a prove for the following statement may take forever:

\[ \exists x \text{ bigger}_\text{than}(x,x) \]
Some curses of FOL (cont.)

• Even if a prover will find a prove, it may take an unreasonable amount of time

• How do we encode all the world knowledge with first order logic?

• There are some more curses - but this talk won’t provide any solution for them :-)}
Description Logic

• A decidable fragment of FOL

• Efficient reasoners (RACER) exist

• Some big knowledge bases are already encoded in description logics (like OWL e.g.)

• We won’t look at a special DL now, but introduce some elements they all have in common
Description Logic - basics

• Designed for knowledge representations

• allowing to encode general knowledge (as above) as well as world models (with individuals, s.a. john)
Description Logic - basics (cont.)

• T-Box: The world‘s rules (as described in the knowledge base)

\[
\begin{align*}
\text{man} & \sqsubseteq \text{person} \\
\text{woman} & \sqsubseteq \text{person} \\
\text{city} & \sqsubseteq \text{location} \\
\forall \text{located_in}.\text{location} \\
\ldots
\end{align*}
\]

• A-Box: Relations between and properties of individuals

\[
\begin{align*}
\text{person(mary)} & \quad \text{works_for(mary, c1)} \\
\text{person(john)} & \quad \text{located_in(NY, c1)} \\
\text{loves(mary, john)} & \quad \text{woman(mary)} \\
\text{loves(john, mary)} & \quad \text{man(john)}
\end{align*}
\]
Description Logic - Terms

• (atomic) concepts $C$ denoting sets of individuals (person) 
  $\approx$ unary predicates in FOL

• (atomic) roles $R$: (loves) $\approx$ binary predicates in FOL

• complex concepts:
  
  • conjunction and disjunction of concepts: $C_1 \sqcap C_2$, $C_1 \sqcup C_2$

  • negation (the complementary concept): $\neg C$

  • existential restriction: $\exists R. C$ (set of all $a$ having an $x$ s.t. $R(a,x) \& C(x)$)

  • value restriction: $\forall R. C$ (set of all $a$ s.t. for all $x$ s.t. $R(a,x), C(x)$ holds)
Description Logic - Terms (cont.)

• inverse roles $R^{-1}$: loves(john, mary) $\equiv$ loves$^{-1}$(mary, john)

• the empty concept $\bot$ and the universal concept $\top$

• concept equality: $C_1 \equiv C_2$
(abbreviates $C_1 \sqsubseteq C_2 \land C_2 \sqsubseteq C_1$)

• `at most` and `at least` number restrictions:
$\exists \leq m R$: Set of all $a$ s.t. there are at most $m$ (different) $x$ for which $R(a,x)$ holds
Description Logic - Example

**A-BOX**

<table>
<thead>
<tr>
<th>Man (john)</th>
<th>Loves (john, mary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Woman (mary)</td>
<td>Loves (mary, sam)</td>
</tr>
<tr>
<td>Man (sam)</td>
<td>Married (sam, sue)</td>
</tr>
<tr>
<td>Woman (sue)</td>
<td>Happy (sam)</td>
</tr>
</tbody>
</table>

Some assertions...

**T-BOX**

<table>
<thead>
<tr>
<th>Bachelor (\equiv \neg \exists \text{married. } \top \sqcap \text{man} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Married (\equiv \text{married}^{-1} )</td>
</tr>
<tr>
<td>(\exists \text{married. } \top \sqsubseteq \text{happy} )</td>
</tr>
<tr>
<td>(\exists \geq 2 \text{love} \sqsubseteq \bot )</td>
</tr>
<tr>
<td>(\exists \text{married. woman} \sqsubseteq \exists \text{love. woman} )</td>
</tr>
</tbody>
</table>

...and some rules:

"bachelors are unmarried men"

(being married to so. is reflexive)

"all married people are happy"

"you can love at most one person"

"someone married to a woman also loves a woman"
Description Logic - RACER

• a reasoner for description logic

• provides reasoning with T-Boxes and (multiple) A-Boxes

• performs consistency checks (of A-Boxes, T-Boxes or both)

• several retrieval tasks:
  • all individuals of a concept, all concepts of an individual
  • check for subsumption („are cities locations?“)
• several retrieval tasks:

  • find the *parent concepts* parents of C are the most specific C‘ s.t. C ⊑ C‘ (children analogously)

  • find *predecessors* (successors): predecessors of C are all C‘ s.t. C ⊑* C‘ (successors analogously)

  • determine *domain* and *fillers* of a role:
    - *fillers* of R are all f s.t. ∃x.R(x,f) (⇔ ∃R⁻¹. ⊤)
    - *domain* of R consists of all d s.t. ∃x.R(d,x) (⇔ ∃R. ⊤)
Example queries:

Is Sue happy?  
(Does 'happy' contain Sue?)

Can Mary love John?  
(loves(mary, john) -> consistent?)

What properties does Mary have?  
(Concepts containing mary)

**A-BOX**

<table>
<thead>
<tr>
<th></th>
<th>loves(john,mary)</th>
<th>loves(mary,sam)</th>
<th>married(sam,sue)</th>
<th>happy(sam)</th>
</tr>
</thead>
<tbody>
<tr>
<td>man(john)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>woman(mary)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>man(sam)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>woman(sue)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**T-BOX**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>bachelor</td>
<td>⊨ ¬ ∃married. T ⊓ man</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>married</td>
<td>⊨ married⁻¹</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∃married. T ⊑ happy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∃≥2 love ⊑ ⊥</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∃married.woman ⊑ ∃love.woman</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
What about Aristotle?

• What’s needed to answer the question whether or not Aristotle is mortal?
What about Aristotle?

• What’s needed to answer the question whether or not Aristotle is mortal?

\[
\text{human(Aristotle)} \sqsubseteq \text{mortal}
\]

A-BOX

human(Aristotle)

T-BOX

human $\sqsubseteq$ mortal

Aristotle $\in$ mortal?
References
