Homework 5
Due: 22nd April 2013, 11:59PM

Q1. (Levin et. al. Ex 5.1, page 73) Show that when \((X_t, Y_t)\) is a coupling satisfying the condition that if \(X_s = Y_s\) then \(X_t = Y_t, t \geq s\), for which \(X_0\) follows the distribution \(\mu\) and \(Y_0\) is distributed as \(\nu\), then

\[
\|\mu P^t - \nu P^t\|_{TV} \leq P(\tau_c > t),
\]

where \(\tau_c\) is the coupling time of the two chains. Further, use this result to give an alternate proof of the convergence theorem.

Q2. (Source: Dana Randall’s class on Markov Chain Monte Carlo Methods, CS8803, 2010) Consider two positive integers \(n, k\) with \(k \leq n/2\). Let \(\Omega\) be the set of all subsets of \(\{1, 2, \ldots, n\}\) of size \(k\). We define a lazy Markov chain on \(\Omega\) as follows: Given a state \(S\), with probability 1/2 do nothing. Otherwise, pick at random \(a \in S\) and \(b \in \{1, 2, \ldots, n\} \setminus S\) and move to state \(S \setminus \{a\} \cup \{b\}\).

1. What is the stationary distribution of this Markov chain?

2. Use a coupling argument to upper bound \(t_{\text{mix}}\) or \(t_{\text{mix}}(\epsilon)\).