Homework 2

Due: **22nd March 2013, 11:59PM**

Q1. (Kazdan, Linear Algebra, Spring 2012, Univ. Pennsylvania) Suppose that λ is an eigenvalue of an $n \times n$ matrix and let E_λ be the set of all eigenvectors with the same eigenvalue. Show that E_λ is a linear subspace of $\mathbb{R}^n$.

Q2. (Levin et. al. Ex 1.7, page 18) A transition matrix P is symmetric if $P(x, y) = P(y, x)$ for all $x, y \in \Omega$. Show that if P is symmetric then the uniform distribution on Ω is stationary for P.

Q3. (Levin et. al. Ex 1.8, page 19) Let P be a transition matrix that is reversible with respect to the probability distribution π on Ω. Show that the transition matrix P^2 corresponding to two steps of the chain is also reversible with respect to π.

Q4. (Levin et. al. Ex 1.13, page 19) A direct proof of the uniqueness of the stationary distribution of an irreducible chain can be given starting from the following argument: Given two stationary distributions π_1 and π_2, consider the state $x \in \Omega$ that minimizes $\pi_1(x)/\pi_2(x)$ and show that all y with $P(x, y) > 0$ have $\pi_1(x)/\pi_2(x) = \pi_1(y)/\pi_2(y)$. Argue this and complete the proof.