1. Let \( n \) be a positive integer and let \( G \) be the set

\[
G = \{ k \mid k \text{ is an integer with } 0 < k < n \text{ and } \gcd(k, n) = 1 \}
\]

Prove that \( G \) is a group under operation \( \otimes \) defined as multiplication modulo \( n \).

2. Prove the Chinese remainder theorem using the previous question. In other words prove that:

If \( m \) and \( n \) are positive integers with \( \gcd(m, n) = 1 \), then there are integers \( a \) and \( b \) such that \( am + bn = 1 \).

3. Let us define a group with two generators \( \{a, b\} \) and let us say that the following relations hold \( ab = b^2a \) and \( ba = a^3b \).

(a) Reduce \( aba^{-1}b^{-1} \) to a string of length 1.
(b) Reduce \( bab^{-1}a^{-1} \) to a string of length 2.
(c) Prove that \( b = a^{-2} \).