1. Recall the definition of the $d$-dimensional cube from the previous homework: For some natural number $d$, let’s say the vertex set of a graph is labelled with the strings from $\{0,1\}^d$ i.e. each vertex has a unique label which is a $d$-bit string and every $d$-bit string corresponds to a vertex. Further we say that there’s an edge between two vertices if their labels differ in exactly one position.

Determine the connectivity of the $d$-dimensional cube.

2. A vertex colouring of a graph is an assignment of colours (or just natural numbers) to the vertices of a graph. A colour class in a coloured graph is a maximal set of vertices which have been assigned the same colour. A graph is said to be $(k,d)$-colourable if the vertices can be coloured with $k$ colours in such a way that the vertices in each colour class induce a graph of maximum degree $d$. Prove the following theorem due to Lovász:

**Theorem 8.1 (Lovász, 1966)** For any $k$, any graph of maximum degree $\Delta$ with $|E|$ edges can be $(k,\lfloor \Delta/k \rfloor)$-coloured in time $O(\Delta|E|)$. 