1. Find a recurrence relation for the number of ways to completely cover a $2 \times n$ chessboard with a $1 \times 2$ dominos. Solve the recurrence to determine this number of ways.

2. Find a recurrence relation for the number of strictly increasing sequences of positive integers that have 1 as their first term and $n$ as their last term where $n$ is a positive integer. That is, sequences $a_1, a_2, \ldots, a_k$ where $a_1 = 1$ and $a_k = n$ and $a_j < a_{j+1}$ for $j = 1, 2, \ldots, k-1$. What are the initial conditions? How many such sequences are there when $n$ is a positive integer and $n \geq 2$?

3. Let $S(m, n)$ denote the number of onto functions from a set with $m$ elements to a set with $n$ elements. Show that $S(m, n)$ satisfies the recurrence:

$$S(m, n) = n^m - \sum_{k=1}^{n-1} \binom{n}{k} S(m, k)$$

whenever $m > n$ and $n > 1$ with the initial condition that $S(m, 1) = 1$. 