1. Find the fallacy in the proof of the following “theorem.”

**Theorem 1.1** A symmetric and transitive binary relation is an equivalence.

**Proof.** Let $\mathcal{R}$ be a symmetric and transitive binary relation on a set $A$. For any pair of elements $(a, b) \in \mathcal{R}$, it follows from symmetry that $(b, a) \in \mathcal{R}$. Further, from transitivity it follows that if $(a, b)$ and $(b, a)$ are in $\mathcal{R}$ then $(a, a)$ and $(b, b)$ are in $\mathcal{R}$. Hence $\mathcal{R}$ is also reflexive and therefore it is an equivalence. $\square$

2. Can you prove that there exists no bijection between $\mathbb{N}^\omega$ and $\mathbb{N}$?

3. Given any preorder $\mathcal{R}$ on a set $A$, prove that the *kernel* of the preorder defined as $\mathcal{R} \cap \mathcal{R}^{-1}$ is an equivalence relation.

4. Consider any preorder $\mathcal{R}$ on a set $A$. We give a construction of another relation as follows. For each $a \in A$, let $[a]_\mathcal{R}$ be the set defined as $\{b \in A | a R b$ and $b R a\}$. Now consider the set $B = \{[a]_\mathcal{R}| a \in A\}$. Let $\mathcal{S}$ be a relation on $B$ such that for every $a, b \in A$, $[a]_\mathcal{R} S [b]_\mathcal{R}$ if and only if $a R b$. Prove that $\mathcal{S}$ is a partial order on the set $B$. 

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