1. Let $a_r$ denote the number of subsets of $\{1, 2, \ldots, r-1, r\}$ which do not contain two consecutive numbers. Determine $a_r$.

2. There are two types of particles inside a nuclear reactor. In every second an $\alpha$ particle will split into three $\beta$ particles and every $\beta$ particle will split into an $\alpha$ particle and two $\beta$ particles. If there is a single $\alpha$ particle at time $t = 0$ then how many particles are there in all at time $t = 100$?

3. Solve the following difference equations:

   (a) $a_r^2 - 2a_{r-1}^2 = 1$, given that $a_0 = 2$.

   (b) $a_r^2 - 2a_{r-1} = 0$, given that $a_0 = 4$.

   (c) $a_r = \sqrt{a_{r-1} + \sqrt{a_{r-2} + \sqrt{a_{r-3} + \cdots}}}$, given that $a_0 = 4$. 