1. **(4.1, Prob 3)** Use induction to prove that

\[
1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}
\]

2. **(4.1, Prob 6)** Prove that

\[
\sum_{i=j}^{n} \binom{i}{j} = \binom{n+1}{j+1}
\]

3. **(4.1, Prob 7)** Prove that every number greater than 7 is a sum of a nonnegative integer multiple of 3 and a nonnegative integer multiple of 5.

4. **(4.1, Prob 7)** Prove by induction that the number of subsets of an \(n\)-element set is \(2^n\).

5. For this problem we need some definitions.

**Definition 2.1** A relation \(R\) on a set \(A\) (i.e. a subset of \(A \times A\)) is called a partial order if it is

- reflexive: \(\forall a \in A : (a, a) \in R\).
- antisymmetric: \(\forall a, b \in A : ((a, b) \in R \land (b, a) \in R) \rightarrow a = b\).
- transitive: \(\forall a, b, c \in A : ((a, b) \in R \land (b, c) \in R) \rightarrow (a, c) \in R\).

A partial order is denoted \(\preceq\) and a partially ordered set or poset \(A\) with a partial order on it is denoted \((A, \preceq)\).

**Definition 2.2** Let \((A, \preceq)\) be a poset. If every two elements of \(A\) are comparable then \(\preceq\) is called a total order and \(A\) is called a totally ordered set.

**Definition 2.3** A poset \((A, \preceq)\) is called a well-ordered set (and \(\preceq\) is called a well-order) if \((A, \preceq)\) is a totally ordered set and every nonempty subset of \(A\) has a least element.

Now the exercise. The following theorem is known as The Principle of Well-Ordered Induction. Prove it.
Theorem 2.1 Given a well-ordered set \((A, \leq)\) and a predicate \(p(x)\), suppose that \(A\) is non-empty and \(x_0\) is the least element of \(A\). Show that if

- Basis: \(p(x_0)\) is true and
- Induction step: For every \(y \in A\), if \(p(x)\) holds for all \(x \leq y\) then \(p(y)\) holds.

Then \(p(x)\) holds for all \(x \in A\).