1. Write the following graph properties as logical statements. Use the predicate \text{in}(x, Y) to denote set membership, \text{subset}(A, B) and also \text{eq}(a, b) which returns \text{T} if \(a\) and \(b\) are the same. Don’t use this to compare numbers. For numbers, use only \text{tt isOne}(n) which returns \text{T} if \(n\) is 1. Use the symbol \(V\) to denote the vertex set of a graph, and \(E\) to denote the edge set of the graph. Also, use the predicate \text{adjacent}(u, v) which applies to vertices \(u\) and \(v\) and returns \text{T} if there is an edge between \(u\) and \(v\).

(a) Minimum degree of the graph is 2.
(b) The graph is not a complete graph.
(c) The graph is triangle-free (i.e. there is no 3-cycle in the graph.)
(d) The graph is a star (i.e. one vertex of degree \(n - 1\) connected to \(n - 1\) vertices of degree 1.)
(e) There is a path of length \(k\) between the vertices \(u\) and \(v\).
(f) The graph is bipartite.

2. Count the following:

(a) The number of graphs on \(n\) vertices.
(b) The number of paths of length \(k\) between two vertices \(u\) and \(v\) in a complete graph on \(n\) vertices.
(c) The number of bipartite graphs on \(n\) vertices.

3. For some natural number \(d\), let’s say the vertex set of a graph is labelled with the strings from \(\{0, 1\}^d\) i.e. each vertex has a unique label which is a \(d\)-bit string and every \(d\)-bit string corresponds to a vertex. Further we say that there’s an edge between two vertices if their labels differ in exactly one position. This graph is known as the \(d\)-dimensional cube. Determine the average degree, number of edges, diameter, girth and circumference of this graph. Give proofs of all your claims.