COL863: Special Topics in Theoretical Computer Science
Spectral Graph Theory
II semester, 2017-18
Linear Algebra refresher problem set

Due: At the beginning of class on Thursday, 11 January 2018.

Problem 1.1 (60=10+10+10+10+10+10 marks) *Attempt problems 1 to 6 of Sec 1.9 of [Spielman2015].*

Problem 1.2 (10 marks) *Construct a non-symmetric matrix M with two distinct eigen values μ and ν such that*

$$M\psi = \mu\psi \text{ and } M\phi = \nu\phi$$

But ψ and and ϕ are not orthogonal, i.e.,

$$\psi^T\phi \neq 0$$

Problem 1.3 (30 marks) *Let M be an $n \times n$ real symmetric matrix with largest eigen value λ and corresponding eigen vector x. Assume that x maximises the Rayleigh quotient*

$$\frac{x^T M x}{x^T x}$$

Extend the proof of this fact given in Spielman's notes to prove the Spectral Theorem *for real symmetric matrices, i.e., there exist real numbers $\lambda_1, ..., \lambda_n$ and n mutually orthogonal unit vectors $\psi_1, ..., \psi_n$ and such that ψ_i is an eigenvector of M of eigenvalue λ_i, for each $i, 1 \leq i \leq n$.*