Problem set: Levin, Peres and Wilmer: Exercises 1.6, 1.7, 1.9, 1.12, 1.15, 2.3, 2.9.

Optional Challenge problem. *This problem is not part of the 100 marks for this test, it is only given as a challenge for you.* Let us consider two “double speed” coupon collection methods and try to figure out which is better. A company has created $n$ coupons named $0, 1, \ldots, n - 1$ and they have put created two kinds of coupon containers: red and blue. In the red containers they have placed two coupons which are randomly selected independent of each other (they may even have placed two copies of the same coupon in some of these). In the blue containers they have placed two coupons by the following rule:

Pick a coupon number, let’s say $j$, at random. Place coupons $j$ and $(j + 1) \mod n$ in the container.

Mr R is allowed to buy only red containers, Mr B is allowed to buy only blue containers. Which one of them collects all the coupons first? Note that this question is not well defined. We could be asking which stopping time has lower expectation or we could be asking “Is it true that one of the stopping times is smaller than the other with probability 1? If so, which one?” You have to figure out answers to both these questions, or analyse the setting in some other way.