Problem 1
Prove that for any binary relations $R$ and $S$ on $A$ (i.e. subset of $A \times A$)
1. $(R^{-1})^{-1} = R$
2. $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$
3. $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$

Problem 2
Prove that for binary relations $R, R'$ from $A$ to $B$ and $S, S'$ from $B$ to $C$, if $R \subseteq R'$ and $S \subseteq S'$ then $R \circ S \subseteq R' \circ S'$.

Problem 3
Given $R \subseteq A \times B$ and $S, T \subseteq B \times C$, prove or find an example that disproves
1. $R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$
2. $R \circ (S \cap T) = (R \circ S) \cap (R \circ T)$
3. $R \circ (S \setminus T) = (R \circ S) \setminus (R \circ T)$

Problem 4
Show that a relation $R$ on a set $A$ is
1. antisymmetric if and only if $R \cap R^{-1} \subseteq I_A$.
2. transitive if and only if $R \circ R \subseteq R$.
3. connected if and only if $(A \times A) \setminus I_A \subseteq R \cup R^{-1}$.

Problem 5
Given a relation $R$ on a set $A$, we define a sequence of relations as follows: we say $R_0 = I_A$ and $R_{i+1} = R_i \circ R$. Based on this we define the reflexive transitive closure of $R$ as $R^* = \bigcup_{i \geq 0} R_i$.

1. Prove that $S = R^* \cup (R^*)^{-1}$ and $T = (R \cup R^{-1})^*$ are both equivalence relations.

2. Prove or disprove $S = T$.

Problem 6
Consider any preorder $R$ on $A$. For each $a \in A$ let $[a]_R = \{b \in A : aRb \land bRa\}$. Now let $B = \{[a]_R : a \in A\}$. Define a relation $S \subseteq B \times B$ as follows: $[a]_R S [b]_R$ whenever $aRb$. Show that $S$ is a partial order.
References

http://www.cse.iitd.ernet.in/~sak/courses/ilcs/logic.pdf