Important: The boxed question is to be submitted at the end of class on a plain sheet of paper with your name, entry number and the tutorial sheet number clearly written at the top of the sheet.

Note: Please read all of Chapter 3 of [1] (even the parts not discussed in class) before attempting this sheet. All questions in this sheet are from that book, question numbers and page indicated in brackets refer to the pdf linked on the course page. Please keep in mind that in class I used → for “implies” but in this tut sheet I will use ⇒.

Problem 1 (Prob 8, pp 94)
Use a truth table to show that \((s \lor t) \land (u \lor v)\) is equivalent to \((s \land u) \lor (s \land v) \lor (t \land u) \lor (t \land v)\).

Problem 2 (Prob 12, pp 94)
Find a statement involving only \(\lor, \land\) and \(\neg\) that is equivalent to \(s \iff t\). Try to ensure that your statement has the fewest possible symbols.

Problem 3 (Prob 4, pp 106)
The definition of a prime number is that it is an integer greater than 1 whose only positive integer factors are itself and 1. Find two ways to write this definition so that all quantifiers are explicit. (It may be convenient to introduce a variable to stand for the number and perhaps a variable or some variables for its factors.)

Problem 4 (Theorem 3.2, pp 100)
Here is the statement of a theorem given in [1] written in slightly different terms.

**Theorem 1**
Suppose we have a domain \(D\) and two predicates \(P, Q : D \to \{T, F\}\). Let \(A = \{x \in D : Q(x) = T\}\).
Show that

1. \(\forall x \in A : P(x)\) is logically equivalent to \(\forall x \in D : Q(x) \Rightarrow P(x)\).
2. \(\exists x \in A : P(x)\) is logically equivalent to \(\exists x \in D : Q(x) \land P(x)\).

Write a proof for this. You may read the proof in the book and then write it in your own words.

Problem 5 (Prob 6, pp 106)
Using \(s(x, y, z)\) to be the statement \(x = yz\) and \(t(x, y)\) to be the statement \(x < y\), write a formal statement for the definition of the greatest common divisor of two numbers.

Problem 6 (Prob 10, pp 107)
Rewrite the following statement without any negations. It is not the case that there exists an integer \(n\) such that \(n > 0\) and for all integers \(m > n\), for every polynomial equation \(p(x) = 0\) of degree \(m\) there are no real numbers for solutions.

Problem 7 (Prob 11, pp 107)
Consider the following slight modification of Theorem 3.2. For each part below, either prove that it is true or give a counterexample. Let \(U_1\) be a universe, and let \(U_2\) be another universe with \(U_1 \subseteq U_2\). Suppose that \(q(x)\) is a statement such that \(U_1 = \{x \mid q(x) \text{ is true}\}\).
1. $\forall x \in U_1(p(x))$ is equivalent to $\forall x \in U_2(q(x) \land p(x))$.

2. $\exists x \in U_1(p(x))$ is equivalent to $\exists x \in U_2(q(x) \implies p(x))$.

Problem 8
Each expression below represents a statement about the integers. Using $p(x)$ for “$x$ is prime,” $q(x, y)$ for “$x = y^2$,” $r(x, y)$ for “$x \leq y$,” $s(x, y, z)$ for “$z = xy$,” and $t(x, y)$ for “$x = y$,” determine which expressions represent true statements and which represent false statements.

1. $\forall x \in Z(\exists y \in Z(q(x, y) \lor p(x)))$.
2. $\forall x \in Z(\forall y \in Z(s(x, x, y) \iff q(x, y)))$.
3. $\forall y \in Z(\exists x \in Z(q(y, x)))$.
4. $\exists z \in Z(\exists x \in Z(\exists y \in Z(p(x) \land p(y) \land \neg t(x, y))))$.

References