COL202: Discrete Mathematical Structures. I semester, 2017-18.
Quiz 8
18 September 2017
Maximum Marks: 4

Name	**Ent. No.**

Important: Keep your answer within the boxes. Anything written outside the box will be treated as rough work. Do your rough work on the free space on the flip side of this sheet.

Problem 1 (2 marks)
In Tutorial Sheet 7 we will prove that any σ-algebra defined on a finite set actually forms a lattice. Here we will draw a Hasse diagram of a σ-algebra. Let us consider three coin tosses thrown according to some probability distribution (it can be anything). Let $\mathcal{F}_i$ be the σ-algebra corresponding to the first i coin tosses. Use the following notation for sets: $A_{\text{shortest possible description of what } A \text{ contains}}$, e.g., if a set contains all strings of type TH or HT we will write $A_{\text{TH or HT}}$. Draw the Hasse diagram of $\mathcal{F}_2$. In it put a * on the side of those nodes of the diagram that also appear in $\mathcal{F}_1$.

Continue overleaf

Problem 2 (1 mark)
Suppose we are told that a collection of sets X with the $\subseteq$ partial order forms a lattice. And suppose that $\mu : X \to \mathbb{R}_+ \cap \{0\}$, i.e., a non-negative valued function, is a measure, i.e. $\mu(S_1 \cup S_2) = \mu(S_1) + \mu(S_2)$ if $S_1 \cap S_2 = \emptyset$, then prove that μ is a monotonic function mapping $(X, \subseteq)$ to $(\mathbb{R}_+ \cap \{0\}, \leq)$ where $\leq$ is the natural "less than or equal to" partial order on non-negative real numbers.

Problem 3 (1 mark)
Use the diagram drawn in Problem 1 and the result of Problem 2 to argue, *without any calculation*, the trivial-sounding result that the probability of getting a head (H) when we toss a single coin is $\leq$ the probability of getting a head when we toss two coins, no matter how the coins are tossed.