CSL356 **Due on :** September 24, 2010

Homework III

1. (a) Suppose we are given two sorted arrays $A[1 \dots n]$ and $B[1 \dots n]$ and an integer k. Describe an algorithm to find the k^{th} smallest element in the union of A and B in $O(\log n)$ time. For example, if $k = 1$, your algorithm should return the smallest element of $A \cup B$; if $k = n$, your algorithm should return the median of $A \cup B$.) You can assume that the arrays contain no duplicate elements.

   (b) Now suppose we are given three sorted arrays $A[1 \dots n], B[1 \dots n]$, and $C[1 \dots n]$, and an integer k. Describe an algorithm to find the k^{th} smallest element in $A \cup B \cup C$ in $O(\log n)$ time.

   (c) Finally, suppose we are given a two dimensional array $A[1 \dots m][1 \dots n]$ in which every row $A[i][]$ is sorted, and an integer k. Describe an algorithm to find the k^{th} smallest element in A as quickly as possible. How does the running time of your algorithm depend on m?

2. Let S be a set of n points in the plane. A point p in S is called Pareto-optimal if no other point in S is both above and to the right of p.Describe and analyze an algorithm that computes **all** the Pareto-optimal points in S in $O(n \log n)$ time.

3. Let $M[1 \dots n][1 \dots n]$ be an $n \times n$ matrix in which every row and every column is sorted. Such an array is called totally monotone. No two elements of M are equal.

   (a) Describe and analyze an algorithm to solve the following problem in $O(n)$ time: Given indices i, j, i', j' as input, compute the number of elements of M smaller than $M[i][j]$ and larger than $M[i'][j']$.

   (b) Describe and analyze an algorithm to solve the following problem in $O(n)$ time: Given indices i, j, i', j' as input, return an element of M chosen uniformly at random from the elements smaller than $M[i][j]$ and larger than $M[i'][j']$. Assume the requested range is always non-empty.

   (c) Describe and analyze a randomized algorithm to compute the median element of M in $O(n \log n)$ expected time.