

## Homework I

1. We say that a directed graph is *nice* if for every pair of vertices  $u$  and  $v$  in the graph, either  $u$  is reachable from  $v$  **or**  $v$  is reachable from  $u$ . Give a linear time algorithm to check if a graph is nice.
2. Consider a directed graph  $G$  where every vertex has a *prize* associated with it. Let  $p(v)$  denote the prize at vertex  $v$ . For a vertex  $u$ , define  $\max(u)$  as the maximum value of  $p(w)$  over all vertices  $w$  which are reachable from  $u$ . Give a linear time algorithm which computes  $\max(u)$  for all the vertices  $u$  in  $G$ .
3. Let  $G$  be an undirected graph and fix a vertex  $s$  in  $G$ . For each vertex  $v$ , let  $N(v)$  denote the **number** of shortest paths from  $s$  to  $v$  (i.e., the length of each of these paths must be equal to the distance of  $v$  from  $s$ ). Give a linear time algorithm which computes  $N(v)$  for each vertex  $v$  in  $G$ . Note that you should not output the actual paths.