COL 351

TUTORIAL SHEET 9

1. Let G be a bipartite graph with V_L and V_R denoting the set of vertices on the two sides. Suppose there is a matching M_1 which matches a subset X of vertices in V_L, and there is a matching M_2 which matches a subset Y of vertices in V_R. Show that there is a matching which matches all the vertices in X and Y.

2. An edge coloring of a graph with k colors assigns a color from the set $\{1, 2, \ldots, k\}$ to each edge such that no two edges sharing a common vertex receive the same color. Show that a bipartite graph where each vertex has degree exactly k has an edge coloring. Extend this result by showing that if Δ denotes the maximum degree of a vertex in a bipartite graph, then there is an edge coloring with Δ colors.

3. Suppose we divide the set of 52 playing cards into 13 groups, where each group contains 4 cards. Then show that it is possible to select one card from each group such that the resulting 13 cards have denomination $2, 3, \ldots, 10, J, Q, K, A$.

4. Let G be a bipartite graph with n vertices on both sides, and let r be the maximum size of any matching in G. Then show that there is a set S of vertices of V_L such that $N(S)$ has size $|S| - r$. Here $N(S)$ denotes the set of vertices in V_R that have at least one edge to a vertex in S.

5. Consider the following greedy algorithm for finding a matching in a bipartite graph: repeatedly select edges which do not share a common vertex till we cannot add any more edge. In the class, we saw that this algorithm may not give a maximum matching. However, show that if m is the size of the maximum matching in the graph, then this algorithm gives matching of size at least $m/2$.

6. Let M be a matching in a bipartite graph and suppose the shortest length of any augmenting path with respect to M is at least k. Prove that the maximum matching in the graph has at most $|M| + \frac{n}{k+1}$ edges, where n is the number of vertices in the graph.