COL 351

TUTORIAL SHEET 12

1. The directed Hamiltonian Cycle Problem is as follows: given a directed graph G, is there a cycle which contains all the vertices ? Suppose you have a polynomial time algorithm for this problem. Show that you can also find such a cycle (if it exists) in polynomial time.

2. The undirected Hamiltonian Cycle Problem can be defined similarly as above. The undirected Hamiltonian Path problem is as follows: given an undirected graph G, is there a path which contains all the vertices ? Show that the undirected Hamiltonian path is polynomial time reducible to the undirected Hamiltonian Cycle problem.

3. [**KT-Chapter8**] Consider a set $A = \{a_1, ..., a_n\}$ and a collection $B_1, B_2, ..., B_m$ of subsets of A. (That is, $B_i \subseteq A$ for each i.) We say that a set $H \subseteq A$ is a hitting set for the collection $B_1, B_2, ..., B_m$ if H contains at least one element from each B_i ? that is, if $H \cap B_i$ is not empty for each i. We now define the Hitting Set problem as follows. We are given a set $A = \{a_1, ..., a_n\}$, a collection $B_1, B_2, ..., B_m$ of subsets of A, and a number k. We are asked: is there a hitting set $H \subseteq A$ for $B_1, B_2, ..., B_m$ so that the size of H is at most k? Prove that Hitting Set is NP-complete.

4. [**KT-Chapter8**] You have a set of friends F whom you're considering to invite, and you're aware of a set of k project groups, $S_1, \ldots, S_k$, among these friends (these sets need not be disjoint). The problem is to decide if there is a set of n of your friends whom you could invite so that not all members of any one group are invited. Prove that this problem is NP-complete.

5. [**KT-Chapter9**] Give an algorithm for the Hamiltonian path problem in a directed graph whose running time is $O(2^n p(n))$, where $p(n)$ is a polynomial in n (here, n denotes the number of vertices in the graph).

6. [**KT-Chapter8**] Consider the following problem. You are given positive integers $x_1, \ldots, x_n$, and numbers k and B. You want to know whether it is possible to partition the numbers $\{x_i\}$ into k sets $S_1, \ldots, S_k$ so that the squared sums of the sets add up to at most B:
$$\sum_{i=1}^{k} \left(\sum_{x_j \in S_i} x_j \right)^2 \leq B.$$
Show that this problem is NP-complete.

7. [**KT-Chapter8**] Given an undirected graph $G = (V, E)$, a feedback set is a set $X \subseteq V$ with the property that $G - X$ has no cycles. The undirected feedback set problem asks: given G and k, does G contain a feedback set of size at most k? Prove that the undirected feedback set problem is NP-complete.