COL 351

TUTORIAL SHEET 1

1. (KT-Chapter 4) Let us consider a long, quiet country road with houses scattered very sparsely along it. (We can picture the road as a long line segment, with an eastern endpoint and a western endpoint.) Further, let's suppose the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road, so that every house is within 4 kilometers of one of the base stations. Give an efficient algorithm that achieves this goal, using as few base stations as possible. Prove the correctness of your algorithm.

2. (KT-Chapter 4) Consider the following variation on the Interval Scheduling Problem from lecture. You have a processor that can operate 24 hours a day, every day. People submit requests to run daily jobs on the processor. Each such job comes with a start time and an end time; if the job is accepted to run on the processor, it must run continuously, every day, for the period between its start and end times. (Note that certain jobs can begin before midnight and end after midnight; this makes for a type of situation different from what we saw in the Interval Scheduling Problem.)

   Given a list of n such jobs, your goal is to accept as many jobs as possible (regardless of their length), subject to the constraint that the processor can run at most one job at any given point in time. Provide an algorithm to do this with a running time that is polynomial in n, the number of jobs. You may assume for simplicity that no two jobs have the same start or end times.

   Example: Consider the following four jobs, specified by (start-time, end-time) pairs: (6 pm, 6 am), (9 pm, 4 am), (3 am, 2 pm), (1 pm, 7 pm). The unique solution would be to pick the two jobs (9 pm, 4 am) and (1 pm, 7 pm), which can be scheduled without overlapping.

3. You are given a line with n points, labeled 1 to n, marked on it. You are also given a set of intervals $I_1, \ldots, I_k$, where interval I_i is of the form $[s_i, e_i]$, $1 \leq s_i \leq e_i \leq n$. Find a set of points X of smallest cardinality such that each interval contains at least one point from X.

4. You are given two sets X and Y of n positive integers each. You are asked to arrange the elements in each of the sets X and Y in some order. Let x_i be the i^{th} element of X in this order, and define y_i similarly. Your goal is to arrange them such that $\Pi_{i=1}^{n} x_i^{y_i} = x_1^{y_1} \times x_2^{y_2} \times \cdots \times x_n^{y_n}$ is maximized. Give an efficient algorithm to solve this problem. Prove correctness of your algorithm.

5. Suppose you want to go from city A to city B on a long highway. Once you fill your car tank to full capacity, it can travel D kilometres. There are several locations on the highway which have petrol pumps. Assume that there is a petrol pump at the start of the highway, and every stretch of length D on the highway has at least one location with a petrol pump. Given the location of these petrol pumps, devise a strategy for traveling from A to B so that you will have to make as few stops for filling petrol as possible.

6. (Jeff Erickson, Algorithms) Consider the following greedy algorithms for the interval scheduling problem discussed in class. Which of these give an optimal solution?

   - If no two intervals overlap, pick all of them. Otherwise, select the interval which overlaps with the fewest number of other intervals. Remove all intervals overlapping with this selected interval and recurse.
   - Select the interval with the highest starting time. Remove all intervals overlapping with this selected interval and recurse.
   - Select the interval with the highest ending time. Remove all intervals overlapping with this selected interval and recurse.
   - If there are two intervals I_1, I_2 such that I_2 is completely contained inside I_1, then discard I_1 and recurse. Otherwise choose the interval which has the highest ending time. Remove all intervals overlapping with this selected interval and recurse.