1. Suppose you have algorithm $A$ which given a graph $G$ and a number $k$, outputs YES iff $G$ has a vertex cover of size at most $k$. Assuming that $A$ runs in polynomial time, show that you can find a vertex cover of minimum size in polynomial time.

2. The directed Hamiltonian Cycle Problem is as follows: given a directed graph $G$, is there a cycle which contains all the vertices? Suppose you have a polynomial time algorithm for this problem. Show that you can also find such a cycle (if it exists) in polynomial time.

3. The undirected Hamiltonian Cycle Problem can be defined similarly as above. The undirected Hamiltonian Path problem is as follows: given an undirected graph $G$, is there a path which contains all the vertices? Show that the undirected Hamiltonian path is polynomial time reducible to the undirected Hamiltonian Cycle problem.

4. Show that the undirected Hamiltonian cycle problem is reducible to the directed Hamiltonian cycle problem. Show that the directed Hamiltonian cycle problem is reducible to the undirected Hamiltonian cycle problem.