1. Show with the help of Fermat’s little theorem that if $n$ is a positive integer, then 42 divides $n^7 - n$.

2. Show that the system of congruences $x \equiv a_1 \pmod{m_1}$ and $x = a_2 \pmod{m_2}$ where $a_1, a_2, m_1$ and $m_2$ are integers with $m_1, m_2 > 0$ has a solution if and only if $a_1 - a_2$ divides $\gcd(m_1, m_2)$.

3. Use the Chinese remainder theorem to show that an integer $a$, with $0 \leq a < m = m_1m_2\ldots m_n$, where the positive integers $m_1, \ldots, m_n$ are pair-wise relatively prime, can be represented uniquely by the $n$-tuple $(a \mod{m_1}, a \mod{m_2}, \ldots, a \mod{m_n})$.

4. Show that if $ac \equiv bd \pmod{m}$ then $a \equiv b \pmod{(m/d)}$, where $d = \gcd(a, b)$.

5. Show that if $a$ and $b$ are positive irrational numbers such that $1/a + 1/b = 1$, then every positive integer can be uniquely expressed as either $\lfloor ka \rfloor$ or $\lfloor kb \rfloor$ for some positive integer $k$.

6. Show that every integer greater than 11 can be written as sum of two composite integers. Recall that a composite integer is a positive integer which is not prime and is larger than 1.

7. Prove that if $f(x)$ is a nonconstant polynomial with integer coefficients, then there is an integer $y$ such that $f(y)$ is composite.

8. A routing transit number (RTN) is a bank code used in the United States which appears on the bottom of checks. The most common form of an RTN has nine digits, where the last digit is a check digit. If $d_1d_2\ldots d_9$ is a valid RTN, then it must be the case that

$$3(d_1 + d_4 + d_7) + 7(d_2 + d_5 + d_8) + (d_3 + d_6 + d_9) = 0 \pmod{10}.$$ 

- Show that if $d_1d_2\ldots d_9$ is a valid RTN, then $d_9 = 7(d_1 + d_4 + d_7) + 3(d_2 + d_5 + d_8) + 9(d_3 + d_6) \pmod{10}$.
- Show that the check digit of an RTN can detect all single errors, and determine which transposition errors an RTN check digit can detect and which ones it cannot detect.