CS130N Problem set 7: Basic graph traversal

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1. Describe the details of an $O(n + m)$ time algorithm for computing all connected components of an undirected Graph G with m vertices and n edges.

2. Let T be a spanning tree produced by the DFS of a connected undirected graph G. Argue why every edge of G, not in T, goes from a vertex v in T to one of its ancestors, that is, it is a back edge.

3. Show that, if T is a BFS tree produced for a connected graph G, then, for each vertex v at level i, the path of T between s and v has i edges, and any other path of G between s and v has at least i edges.

4. Given a tree T of n nodes the diameter of T is the length of a longest path between two nodes of T. Give an efficient algorithm to compute the diameter of T.

5. An independent set of an undirected graph $G = (V, E)$ is a subset I of V such that no two vertices in I are adjacent. That is, if $u, v \in I$ then $(u, v) \notin E$. A maximal independent set M is an independent set such that, if we were to add any additional vertex to M, then it would not be independent any more. Prove that every graph has a maximal independent set. Give an efficient algorithm to compute the maximal independent set of a given graph G.

6. An *Euler* tour of a directed graph G with n vertices and m edges is a cycle that traverses each edge of G exactly once according to its direction. Such a tour always exists if the in-degree equals the out-degree for every vertex in G. Describe an $O(n + m)$ time algorithm for finding a Euler tour of such a graph G.

7. Let G be an undirected graph with n vertices and m edges. Describe an $O(n+m)$ time algorithm for traversing each edge of G exactly once in each direction.

8. A node p of a directed graph $G = (V, E)$ is called a *sink* if for every node $v \in V, v \neq p$ the edge (v, p) exists, whereas the edge (p, v) does not exist. Describe an algorithm that can detect the presence of a sink in G in $O(n)$ time (n is the number of vertices). Your algorithm should accept the graph represented by its adjacency matrix. (Notice that the running time $O(n)$ for this problem is remarkable given that the instance takes a space in $\Omega(n^2)$ merely to write down).