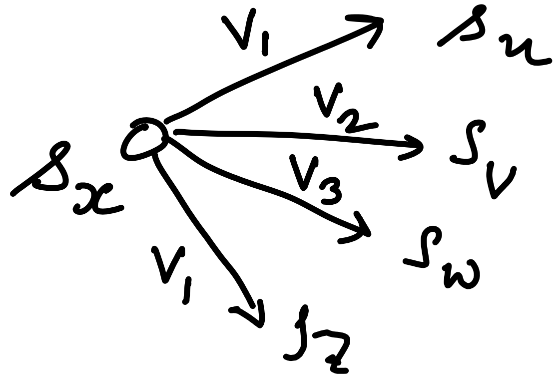
CSL 356 Lecture 22, Sept 24

In class quiz on Fri (Sept 27)
syllabus - dynamic Prog

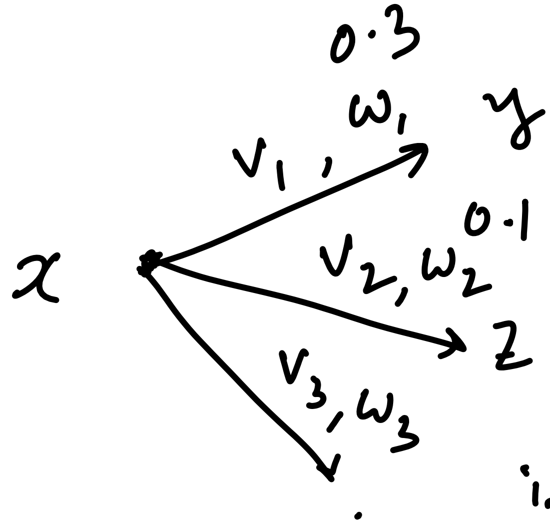
Often for many real life applications we form hypotheses on the basis of observations and we would like to form a hypothesis that is most likely.

We have a weighted directed graph, say $G = (V, E, W)$
W is a weight function

Vertices represent the "states" corresponding to some partial deduction about our hypotheses



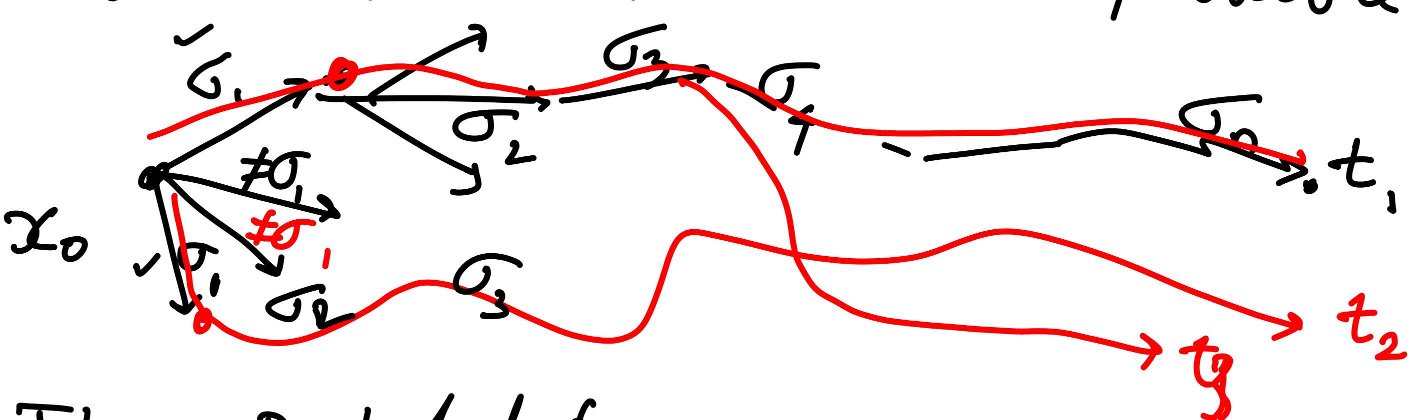
depending on the observation, say O that can have several values $v_1, v_2, \ldots$



w_i corresponds to our confidence in moving to the next state. In fact if we normalize the weights, we can make them correspond to probabilities

Given observations $\overbrace{\sigma_1 \sigma_2 \sigma_3 \ldots}^{\sigma} \sigma_n$

σ_i is an observation, ~~Start~~ Starting from an initial state (some starting vertex), we want to look at all the possible paths in the graph corresponding to σ and choose the one that is most probable



The probability of a specific path

$$P = e_1 e_2 e_3 \ldots e_n = \prod_i w(e_i)$$

Take the logarithm of the prod of probabilities

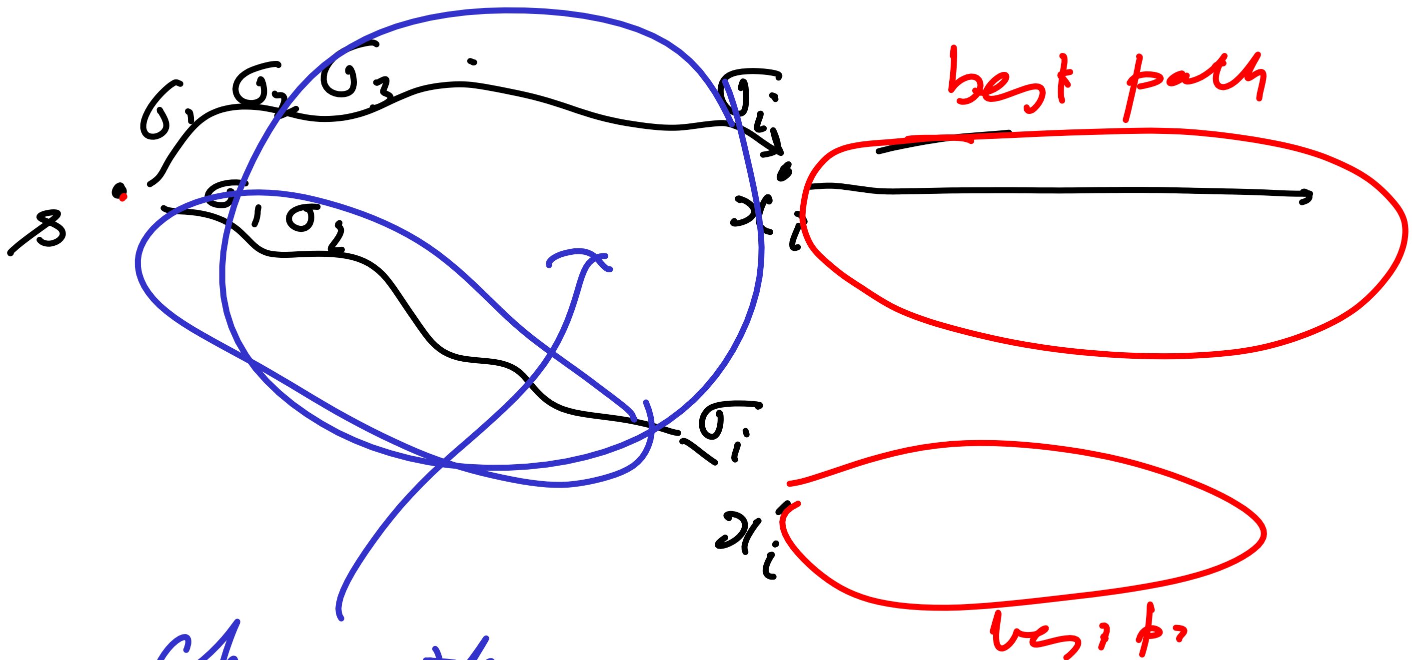
$$\log \left(\underbrace{w(e_1) \cdot w(e_2) \cdot w(e_3)}_{\text{correspond to prob}} \cdots w(e_n) \right)$$

$$\Rightarrow \quad \sum \underbrace{\log(w(e_1))}_{w'} \cdots$$

Since logarithms of prob are negative we will actually minimize the sum of the weights (that are logs)

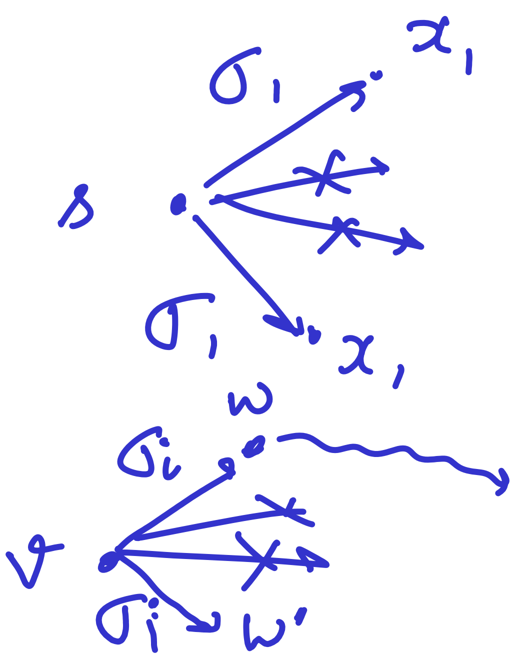
<u>Objective</u>. Given a labelled, weighted graph $G = (V, E, W)$; ... and a sequence σ of labels $\sigma_1\ \sigma_2\ \sigma_3 \cdots \sigma_n$, we want to find the path $P^* = e_1^* e_2^* e_3^* \cdots e_n^*$ such that $\sum_i w(e_i^*)$ is minimised where e_i^* has the label σ_i^*

$\sigma_1 \; \sigma_2 \; \sigma_3 \; \sigma_i \; \cdot \; \sigma_n$



Choose the best path

Choose the best intermediate point



Let $P_i(v)$ denote the best path from vertex v with labels $\sigma_{i+1}\sigma_{i+2}\cdots\sigma_n$

$$P_{i-1}(v) = \min_{\substack{w \cdot| \\ \text{label}(v,w)=\sigma_i}} \left[P_i(w) + W(v,w) \right]$$

Must be calculate for all vertices $v \in V$

Eventually or finally, we need

$P_0(s)$. Base case paths of length 1 namely

$P_{n-1}(v)$ $\forall v \in V$

Running time? : We need to compute $P_i(v)$ for $0 \leq i \leq n$, $v \in V$

$\Rightarrow$ $O(n \cdot |V|)$ terms

How much time for each? : degree of a vertex which is $O(|V|)$

$\Rightarrow$ Total time $O(n\,|V|^2)$

Space : If all terms have to be stored, then $O(n|V|)$

<u>Viterbi's algorithm</u>