CSL 356 Lec 21 Sept 17

Finding longest monotonic subsequence in an array of n numbers – $x_1, x_2 \ldots x_n$

1. Dynamic prog formulation

S_i : Longest mono sequence ending in x_i

$|S_i|$: length

Then $|S_k| = \max_{1 \le i < k} \{ |S_i| + 1 \mid x_i \le x_k \}$ $k \le n$

if $x_i > x_k \; \forall k$, then 1

$S_1 = x_1$

Interested in S_n

<u>Analysis</u> : Time : $O(k)$ for k^{th} term

$\Rightarrow O(n^2)$ for S_n

Space : $O(n)$ (all previous terms may be required)

Improving to $O(n \log n)$

$M_{i,j}$: denotes the subsequence of mono incr. of length j in if one exists

$x_1, x_2 \dots x_i$

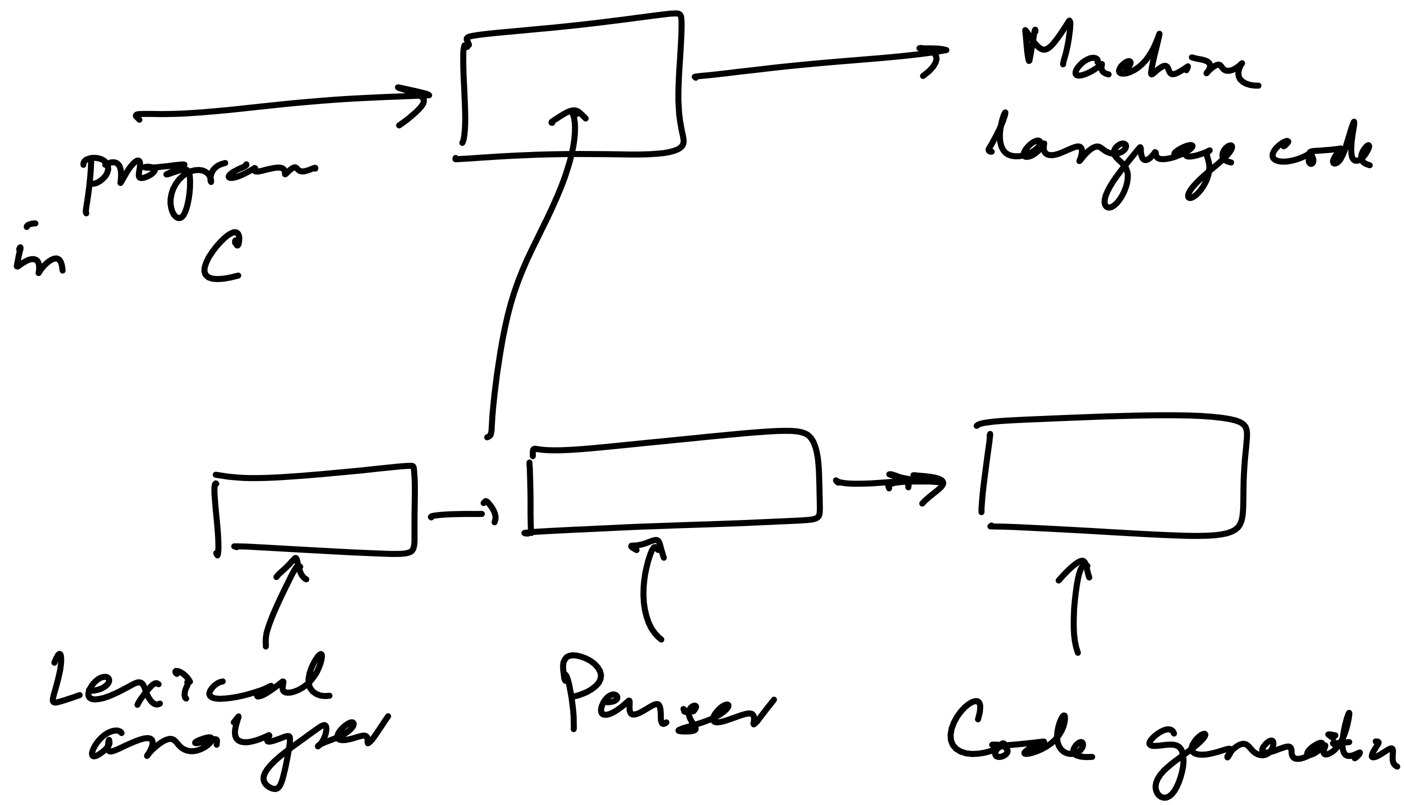
that has the "smallest" last term"

2, 5, 8, 9, 11

2 7 8, 9, 10 ✓

$\max_j \{ M_{n,j} \}$

Compiler



grammar of the language must be adhered to

1. $S \to AB$
2. $S \to BC$
3. $A \to BA$
4. $A \to a$
5. $B \to CC$
6. $B \to b$
7. $C \to AB$
8. $C \to a$

S, A, B, C : Variables / non-terminals

a, b, c : terminals

$baaba \in$ Gram.

$S \xrightarrow{(1)} AB \xrightarrow{(4)} aB \xrightarrow{(5)} aCC \xrightarrow{8,8} aaa$

$S \xrightarrow{*} aaa$ aaa belongs to the grammar described above

Context Free Grammar
in Chomsky Normal Form (CNF)

Compiler has to solve the reverse problem, i.e. given a string S, say of length n, can we produce S from the grammar?

If so, correct, produce code

If not,

Is the string baaba in the above grammar?

If baaba belongs to the grammar then there must be a derivation

$$S \xrightarrow{*} baaba$$

$$S \to \underset{1}{AB} \xrightarrow{*} \underbrace{ba}_{A}\underbrace{aba}_{B}$$

$$A \xrightarrow{*}$$

$$S \to BC \xrightarrow{*} \underbrace{ba}_{B}\underbrace{aba}_{C}$$

$$S \underset{?}{\xrightarrow{*}} baaba$$

1st pos: $A \xrightarrow{*} ba$ and $B \xrightarrow{*} aba$

2nd pos: $A \xrightarrow{*} a$, $B \xrightarrow{*} aaba$

3rd: $A \xrightarrow{*} baa$, $B \xrightarrow{*} ba$

4th: $A \xrightarrow{*} baab$, $B \xrightarrow{*} a$

All are substrings of the original string b a a b a substrings $O(n^2)$

b ~~a~~ a subsequence $O(2^n)$

Let us denote a substring by $\delta_{i,j}$ that begins from i and has length j

For any symbol, say S, A, B, C..

$$A \xrightarrow{*} \delta_{i,j} \quad \forall i,j \ ?$$

$$S \xrightarrow[?]{*} \delta_{1,n}$$