Linear-time Properties   L08   01 Feb 2010

Given a DTS $\mathcal{D} = \langle S, \longrightarrow, AP, D, I \rangle$ and an infinite sequence $\sigma \in S^\omega$ of states (representing an execution of $\mathcal{D}$) we may extend the decoration function D so as to yield an infinite sequence π of sets of atoms i.e.

$$D : S^\omega \longrightarrow (2^{AP})^\omega$$

with $D(\sigma) = \pi$ so that for any state s_i in the sequence $\sigma = s_0 s_1 s_2 \ldots$,

$D(\sigma) = \pi = P_0 P_1 P_2 \ldots$ with $P_i = D(s_i) \in 2^{AP}$.

Let Σ be a set of infinite sequences of states

Then $D(\Sigma) = \{ D(\sigma) \mid \sigma \in \Sigma \} = \Pi$

We could go further and use $\overline{D}$ instead of D. where for any state s,

$$\overline{D}(s) = D(s) \cup \{ \neg q \mid q \in AP \setminus D(s) \}$$

and $\overline{D}(\sigma) = \overline{\pi} = \overline{P_0}\,\overline{P_1}\,\overline{P_2} \ldots$ where $\overline{P_i} = \overline{D}(s_i)$

and $\overline{\Pi} = \overline{D}(\Sigma)$ where Σ is the set of executions of $\mathcal{D}$.

Invariant properties.

<u>Def</u>. Let Σ be the set of executions of a DTS $\mathcal{D}$. A propositional formula φ over AP is called an invariant property of $\sigma \in \Sigma$, if $\overline{P_i} \Rightarrow \varphi$ for every $i \geq 0$, where $\overline{D}(\sigma) = \overline{\pi} = \overline{P_0}\,\overline{P_1}\,\overline{P_2}\ldots$. Further φ is an invariant of $\mathcal{D}$ if φ is an invariant property of every $\sigma \in \Sigma$.

<u>Claim</u>. A proposition φ is invariant of $\mathcal{D}$ iff it is true of all the reachable states of $\mathcal{D}$.

<u>Note</u>: 1. For any σ, φ is a propositional invariant of σ iff $\sigma \models G\varphi$.

2. φ is a propositional invariant of $\mathcal{D}$ iff $\mathcal{D} \models G\varphi$.