## CS 210N: Numerical and Scientific Computing

## Tutorial - 6

1. For the matrix $\left[\begin{array}{llll}0 & 1 & 4 & 0 \\ 0 & 2 & 8 & 0\end{array}\right]$, determine the Echelon form $U$, the pivot variables and null space. Then apply elimination to $\mathrm{Ax}=\mathrm{b}$, with components $\boldsymbol{b}_{1}$ and $\quad \boldsymbol{b}_{2}$ on the right side; find the condition for $\mathrm{Ax}=\mathrm{b}$ to be consistent (i.e. to have a solution and find the general solution. What is the rank of A?
2. Carry out the above for $\left[\begin{array}{ll}0 & 0 \\ 1 & 2 \\ 4 & 8 \\ 0 & 0\end{array}\right]$.
3. Write the general solution to $\left[\begin{array}{llll}0 & 1 & 4 & 0 \\ 0 & 2 & 8 & 0\end{array}\right]\left[\begin{array}{l}\boldsymbol{u} \\ \boldsymbol{v} \\ \boldsymbol{w}\end{array}\right]=\left[\begin{array}{l}1 \\ 4\end{array}\right]$
4. Describe the set of attainable right sides $b$ for $\left[\begin{array}{ll}1 & 0 \\ 0 & 1 \\ 2 & 3\end{array}\right]\left[\begin{array}{l}\boldsymbol{u} \\ \boldsymbol{v}\end{array}\right]=\left[\begin{array}{l}\boldsymbol{b}_{1} \\ \boldsymbol{b}_{2} \\ \boldsymbol{b}_{3}\end{array}\right]$, by finding the constraints on $b$ that turn the third equation into $0=0$ (after elimination). What is the Rank?
5. What is the most general solution to

$$
\begin{aligned}
& u+\boldsymbol{v}+\boldsymbol{w}=1 \\
& \boldsymbol{u}-\boldsymbol{w}=2
\end{aligned} ?
$$

