CS 210N: Numerical and Scientific Computing

Tutorial – 5

1. Perform four iterations of Newton's method for the polynomial
   $$p(x) = 4x^3 - 2x^2 + 3$$
   Starting with $x_0 = -1$. Use a calculator.

2. To compute reciprocals without division, we can solve ***x = 1/R*** by finding a zero of the function $f(x) = x^{-1} - R$. Write a short algorithm to find ***1/R*** by Newton's method applied to ***f***. Do not use division or exponentiation in your algorithm. For Positive ***R***, what starting points are suitable?

3. If Newton's method is use on $f(x) = x^3 - 2$ starting with $x_0 = 1$, what is x_2?

4. If the secant method is applied to the function $f(x) = x^2 - 2$ with $x_0 = 0$ and $x_1 = 1$, what is x_2?

5. Apply the Newton-Raphson method with $x_0 = 0.8$ to the equation $f(x) = x^3 - x^2 - x + 1 = 0$ and verify that the convergence is only of first order in each case. Then apply the Newton-Raphson method $x_{n+1} = x_n - m\dfrac{f(x_n)}{f'(x_n)}$ with m = 2 and verify that the convergence is of second order.

6. Find a catenary y = c cosh((x – a)/ c) passing through the points (1,1) and (2,3).

7. The equation $x^2 + ax + b = 0$ has two real roots α and β. Show that the iteration method $x_{n+1} = 0(ax_k + b)/x_k$ is convergent near α if $|\alpha| > |\beta|$.

8. Write out Newton's iteration for solving each of the following nonlinear equations:
   (a) $x^3 - 2x - 5 = 0$
   (b) $e^{-x} = x$
   (c) x sin(x) = 1.

9. Obtain the complex roots of the equation $f(z) = z^3 + 1 = 0$ correct to eight decimal places. Compare with the exact values of the roots $\left(1 \pm i\sqrt{3}\right)/2$.

10. What condition ensures that the bisection method will find a zero of a continuous nonlinear function f in the interval [a, b]?