

CS 210N: Numerical and Scientific Computing

Tutorial – 4

1. Which of the following behaviors are possible in using Newton's method for solving a nonlinear equation?

- (a) It may converge linearly
- (b) It may converge quadratically
- (c) It may not converge at all

2. What is meant by a bracket for a non-linear function in one dimension? What does this concept have to do with zero finding?

3. What is meant by a quadratic convergence rate for an iterative method?

4. What is meant by a fixed point of a function $g(x)$?

5. Find the iterative methods based on the Newton Raphson methods for finding square root, inverse and cubic root of a real positive number N . Apply the method to $N = 18$ to obtain the result correct to two decimals.

6. Given the following equations:

(i) $x^4 - x - 10 = 0$ (ii) $x - e^{-x} = 0$

Determine the initial approximations. Use these to find the roots corresponding to three decimal places with the following methods:

- (a) the Regula Falsi method
- (b) the Secant method
- (c) the Newton-Raphson method

7. Perform two iterations with Muller method for the following equations:

(i) $x^3 - 1/2 = 0, x_0 = 0, x_1 = 1, x_2 = 1/2$

(ii) $\log x - x + 3 = 0, x_0 = 1/4, x_1 = 1/2, x_2 = 1$

8. Express the Newton Iteration for solving each of the following nonlinear systems

(a)
$$\begin{aligned} x_1^2 + x_2^2 &= 0 \\ x_1^2 - x_2^2 &= 0 \end{aligned}$$

(b)
$$\begin{aligned} x_1^2 + x_1 x_2^3 &= 9 \\ 3x_1^2 x_2 - x_2^3 &= 4 \end{aligned}$$