# CS 210N: Numerical and Scientific Computing

**Tutorial – 3**

1. In general, which matrix norm is easier to compute, $\|\boldsymbol{A}\|_1$ or $\|\boldsymbol{A}\|_2$? Why?

2. What is the condition number of the following matrix using the 1-norm?

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Does you answer differ using the $\infty$- norm?

3. Is the magnitude of the determinant of a matrix a good indicator of whether the matrix is nearly singular? If so, why? If not, what is a better indicator of near singularity?

4. Let A = diag(1/2) be an n x n diagonal matrix with all its diagonal entries equal to ½.
   (a) What is the value of det(A)?
   (b) What is the value of cond(A)?
   (c) What conclusion can you draw from these results?

5. Classify each of the following matrices as well-conditioned or ill-conditioned:

$$\begin{bmatrix} 10^{10} & 0 \\ 0 & 10^{10} \end{bmatrix} \begin{bmatrix} 10^{-10} & 0 \\ 0 & 10^{-10} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

6. What is the Cholesky factorization of the following matrix

$$\begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$

7. Using infinity norm, compute the condition number of the matrix $\begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix}$.

8. What is the inverse of the following matrix?

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

9. True or False: (a) The norm of a singular matrix is zero, (b) A symmetric positive definite matrix is always well-conditioned.

10. Under what circumstances does a small residual vector $\boldsymbol{r} = \boldsymbol{b} - \boldsymbol{Ax}$ imply that $\boldsymbol{x}$ is an accurate solution to the linear system $\boldsymbol{Ax} = \boldsymbol{b}$?