CS 210N: Numerical and Scientific Computing

Tutorial – 2

1. Solve the following linear systems twice. First, use Gaussian elimination and give factorization A=LU. Second, use Gaussian elimination with scaled row pivoting and determine the factorization of the form PA=LU

(a) $$\begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1/2 \end{bmatrix}$$

(b) $$\begin{bmatrix} 1 & 6 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

© $$\begin{bmatrix} -1 & 1 & 0 & -3 \\ 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & -1 \\ 3 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

(d) $$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 4 & -9 & 2 & 1 \\ 8 & 16 & 6 & 5 \\ 2 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 14 \\ -3 \\ 0 \end{bmatrix}$$

2. Show how Gaussian elimination with scaled row pivoting works on this example (Forward phase only):

$$\begin{bmatrix} 2 & -2 & -4 \\ 1 & 1 & -1 \\ 3 & 7 & 5 \end{bmatrix}$$

Display the scale array (s_1, s_2, s_3) and find the final permutation array (p_1,p_2,p_3). Show the final A-array, with the multipliers stored in the correct locations.

3. This problem shows how the solution of the system of equations can be unstable relative to perturbations in the data. Solve Ax = b with b = $(100, 1)^T$, and with each of the following matrices.

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 0.01 \end{bmatrix}$$

4. Solve this system by Gaussian elimination with full pivoting

$$\begin{bmatrix} -9 & 1 & 17 \\ 3 & 2 & -1 \\ 6 & 8 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ -3 \end{bmatrix}$$

5. Determine det(A), where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

without computing the determinant by expansion by minors.

6. Given an *n x n* nonsingular matrix **A** and a second *n x n* matrix **B**, what is the best way to compute the *n x n* matrix $\mathbf{A}^{-1}\mathbf{B}$?

7. Specify an elementary elimination matrix the zeros the last two components of the vector

$$\begin{bmatrix} 3 \\ 2 \\ -1 \\ 4 \end{bmatrix}$$

8. True or False: The multipliers in the Gaussian algorithm with full pivoting (both row and column pivoting) lie in the interval [-1,1].

9. Consider $A = \begin{bmatrix} 3 & 2 & -1 \\ 6 & 6 & 2 \\ -1 & 1 & 3 \end{bmatrix}$. Use Gaussian elimination with scaled row pivoting to obtain the factorization $PA = LDU$, where L is a Unit lower triangular matrix, U is unit upper triangular matrix, D is a diagonal matrix, and P is a permutation matrix.

10. If **A** is *n x n* and **B** is *n x m*, how many multiplications and divisions are required to solve **AX = B** by Gaussian elimination with scaled row pivoting? What is **B = I** ?