CSL863: Randomized Algorithms
II semester, 2007-08

Homework # 2

Due before class on **Tuesday, 11th March 2008**

Instructor: Amitabha Bagchi

February 19, 2008

A graph picked at random from the set of all graphs is called a *random graph*. There are several models of random graphs, but two models that are popular are denoted $G_{n,p}$ and $G_{n,k}$.

- In $G_{n,p}$ we consider all the graphs on n distinct vertices $v_1, v_2, \ldots, v_n$. To generate these graphs we consider each of the $\binom{n}{2}$ possible edges in some order and then independently add each edge to the graph with probability p. The expected number of edges in $G_{n,p}$ is hence $\binom{n}{2}p$.

- In $G_{n,k}$ we consider all the graphs on n vertices with *exactly* k edges. There are clearly $\binom{\binom{n}{2}}{k}$ such graphs from which one is selected uniformly at random. One way of generating a graph from $G_{n,k}$ is to pick an edge uniformly at random from the $\binom{n}{2}$ possible edges, then pick another one independently of the first from the remaining $\binom{n}{2} - 1$ possible edges and so on till k edges have been picked.

We now try to relate these two models in a manner similar to the manner in which we related the balls and bins setting with the independent Poisson trials model.[1]

P1. Prove that every event that happens with bounded probability in $G_{n,p}$ also happens with bounded probability in $G_{n,k}$ with $k = \binom{n}{2}p$.

P2. Use the result of the previous part to show that there exists a constant c such that if $k \geq cn \log n$ then the probability that a graph chosen from $G_{n,k}$ is connected is $1 - o(1)$. We consider an undirected graph on n vertices disconnected if there exists a set of $k < n$ vertices such that there is no edge between this set of vertices and the rest of the graph.

[1]This relationship is described in Mitzenmacher and Upfal's book. The problems here appear as Ex 5.17 and 5.18 in their text.