CSL857: Randomized algorithms
II Sem, 2012-13, IIT Delhi

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Homework 5

Due: **22nd April 2013, 11:59PM**

Q1. (Levin et. al. Ex 5.1, page 73) Show that when (X_t, Y_t) is a coupling satisfying the condition that if $X_s = Y_s$ then $X_t = Y_t, t \geq s$, for which X_0 follows the distribution μ and Y_0 is distributed as ν, then

$$\|\mu P^t - \nu P^t\|_{TV} \leq \mathrm{P}(\tau_c > t),$$

where τ_c is the coupling time of the two chains. Further, use this result to give an alternate proof of the convergence theorem.

Q2. (Source: Dana Randall's class on Markov Chain Monte Carlo Methods, CS8803, 2010) Consider two positive integers n, k with $k \leq n/2$. Let Ω be the set of all subsets of $\{1, 2, \ldots, n\}$ of size k. We define a lazy Markov chain on Ω as follows: Given a state S, with probability 1/2 do nothing. Otherwise, pick at random $a \in S$ and $b \in \{1, 2, \ldots, n\} \setminus S$ and move to state $S \setminus \{a\} \cup \{b\}$.

1. What is the stationary distribution of this Markov chain?
2. Use a coupling argument to upper bound t_{mix} or $t_{\text{mix}}(\epsilon)$.