CSL857: Randomized algorithms | Instructor: Amitabha Bagchi
II Sem, 2012-13, IIT Delhi | Submitted by: Your name here

Homework 4

Due: **12th April 2013, 11:59PM**

Q1. (Levin et. al. Ex 2.8, page 34) Show that if a random walk on a group is reversible then the increment distribution is symmetric.

Q2. (Levin et. al. Ex 2.10, page 34) Read Section 2.7 of Levin's book which was skipped in class and then solve the following problem. Let $\{S_n\}_{n\geq 0}$ be the simple random walk on $\mathbb{Z}$. Use the reflection principle discussed in Section 2.7 to show that

$$\mathrm{P}\left(\max_{i\leq j\leq n} |S_j| \geq c\right) \leq 2\mathrm{P}(|S_n| \geq c).$$

Q3. (Levin et. al. Ex 4.3 and Ex 4.4, page 59) Let P be the transition matrix of a Markov chain with state space Ω and let μ and ν be any two distributions on Ω. Prove that

$$\|\mu P - \nu P\|_{TV} \leq \|\mu - \nu\|_{TV}.$$

Use this fact, or prove otherwise that for a Markov chain with stationary distribution π, for any $t \geq 0$

$$d(t+1) \leq d(t).$$

Q4. Given a simple random walk on a graph the *hitting time* of vertex j starting from vertex i, denoted $H_{i,j}$, is the expected number of steps taken to reach j starting from i. And the cover time of the random walk starting from node i, denoted C_i, is the expected number of steps taken to visit every node of the graph at least once when the walk is started from node i. The cover times of the random walk, C, is $\max_{i\in V} C_i$.

Q4.1. What is the hitting time of any pair $H_{i,j}$ in the random walk on a complete graph on n nodes?

Q4.2. Compute the cover time of the random walk on the complete graph on n nodes.

Q4.3. Prove that if $H = \max_{i,j\in V} H_{i,j}$ for a random walk on some graph G of size n, then $C \leq 2\log n \cdot H$. (**Hint.** Prove that if b is the time taken to vist at least half the nodes of the graph then $b \leq 2H$.)