CSL857: Randomized algorithms II Sem, 2012-13, IIT Delhi	Instructor: Amitabha Bagchi Submitted by: Your name here

Homework 2

Due: **22nd February 2013, 11:59PM**

Q1. (Billingsley 5.17, page 83)

(a) Suppose that $X_n \to_P X$ and that f is a continuous function. Show that $f(X_n) \to_P f(X)$.

(b) Show that $\mathrm{E}[|X - X_n|] \to 0$ implies $X_n \to_P X$. Show that the converse is false.

Q2. (Billingsley 6.1, page 89) Show that $Z_n \to Z$ with probability 1 if and only if for every positive ϵ there exists an n such that $\mathrm{P}[|Z_k - Z| < \epsilon, n \leq k \leq m] > 1 - \epsilon$ for all m exceeding n. This describes convergence with probability 1 in "finite" terms.

In all the following problems $S_n = X_1 + \cdots + X_n$.

Q3. (Billingsley 6.7, page 90)

(a) Let $x_1, x_2, \ldots$ be a sequence of real number and put $s_n = x_1 + \cdots + x_n$. Assuming that $n^{-2} s_{n^2} \to 0$ and that the x_n are bounded (i.e. each of them is finite), show that $n^{-1} s_n \to 0$.

(b) Suppose that $n^{-2} S_{n^2} \to 0$ with probability 1 and that the X_n are uniformly bounded (i.e. $\sup_{n,\omega} |X_n(\omega)|$ is finite). Show that $n^{-1} S_n \to 0$ with probability 1. Here the X_n need not be identically distributed or independent.

Q4. (Billingsley 6.11, page 90) Suppose that $X_1, X_2, \ldots$ are m-dependent in the sense that random variables more than m apart in the sequence are independent. More precisely, let $\mathcal{A}_j^k = \sigma(X_k, \ldots, X_k)$ and assume that $\mathcal{A}_{j_1}^{k_1}, \ldots, \mathcal{A}_{j_l}^{k_l}$ are independent if $k_{i-1} + m < j_i$ for $i = 2, \ldots, l$. Suppose that X_n have this property and are uniformly bounded and that $\mathrm{E}[X_n] = 0$. Show that $n^{-1} S_n \to 0$.