

CS105L: Discrete Structures  
I semester, 2006-07

Tutorial Sheet 3: Propositional Calculus and Hilbert Systems

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1. Prove in the Hilbert system, using induction, that the following rule (known as the **deduction rule**) is sound:

$$\frac{U \cup \{A\} \vdash B}{U \vdash A \Rightarrow B}.$$

2. Prove the **contrapositive rule** in the Hilbert system using only the axioms and modus ponens i.e. show that

$$\frac{\vdash \neg B \Rightarrow \neg A}{\vdash A \Rightarrow B}.$$

3. Prove in the Hilbert system, using derivation rules if necessary:

- (a)  $\vdash A \Rightarrow \neg\neg A$ .
- (b)  $\vdash (A \Rightarrow \neg A) \Rightarrow \neg A$ .
- (c)  $\vdash A \Rightarrow (B \Rightarrow (A \wedge B))$ .
- (d)  $\vdash A \Rightarrow A \vee B$ .
- (e)  $\vdash (A \Rightarrow B) \Rightarrow ((C \vee A) \Rightarrow (C \vee B))$ .