CS105L: Discrete Structures
I semester, 2006-07

Tutorial Sheet 1: Sets, Relations and Functions

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1. Prove that for any binary relations $\mathcal{R}$ and $\mathcal{S}$ on a set A,
   (a) $(\mathcal{R}^{-1})^{-1} = \mathcal{R}$
   (b) $(\mathcal{R} \cap \mathcal{S})^{-1} = \mathcal{R}^{-1} \cap \mathcal{S}^{-1}$
   (c) $(\mathcal{R} \cup \mathcal{S})^{-1} = \mathcal{R}^{-1} \cup \mathcal{S}^{-1}$
   (d) $(\mathcal{R} \setminus \mathcal{S})^{-1} = \mathcal{R}^{-1} \setminus \mathcal{S}^{-1}$
2. Show that a relation $\mathcal{R}$ on a set A is
   (a) reflexive if and only if $\mathcal{I}_A \subseteq \mathcal{R}$;
   (b) antisymmetric if and only if $\mathcal{R} \cap \mathcal{R}^{-1} \subseteq \mathcal{I}_A$;
   (c) transitive if and only if $\mathcal{R} \circ \mathcal{R} \subseteq \mathcal{R}$
   (d) connected if and only if $(A \times A) \setminus \mathcal{I}_A \subseteq \mathcal{R} \cup \mathcal{R}^{-1}$.
3. Prove that there exists a bijection from $\mathbb{N}^2$ to $\mathbb{N}$, where $\mathbb{N}$ denotes the set of natural numbers.
4. Prove that for any relation $\mathcal{R}$ on a set A
   (a) $\mathcal{S} = \mathcal{R}^* \cup (\mathcal{R}^*)^{-1}$ and $\mathcal{T} = (\mathcal{R} \cup \mathcal{R}^{-1})^*$ are both equivalence relations.
   (b) Prove or disprove: $\mathcal{S} = \mathcal{T}$.