CS105L: Discrete Structures
I semester, 2006-07

Homework # 7

Due before class on **Friday, September 29th, 2006**

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Note: Try to use generating functions to solve problems 1 to 3, even the ones which do not explicitly mention them. It could also be useful to attempt some of the counting or summation problems both with and without generating function.

1. Let a_r denote the number of ways to seat 10 students in r chairs so that no two students sit in adjacent chairs. Determine the generating function of this numeric function.
2. In how many ways can $3r$ balls be chosen from $2r$ red balls, $2r$ blue ballls and $2r$ green balls?
3. Evaluate the following sums:

   (a)
   $$\binom{n}{1} + 2 \cdot \binom{n}{2} + \cdots + i \cdot \binom{n}{i} + \cdots + n \cdot \binom{n}{n}$$

   (b) Given that $k \leq m$ and $k \leq n$
   $$\binom{n}{0} \cdot \binom{m}{k} + \binom{n}{1} \cdot \binom{m}{k-1} + \binom{n}{2} \cdot \binom{m}{k-2} + \cdots + \binom{n}{k} \cdot \binom{m}{0},$$

   (c)
   $$\binom{2n}{n} + \binom{2n-1}{n-1} + \cdots + \binom{2n-i}{n-i} + \cdots + \binom{n}{0}$$

4. Given is a planar set of 25 points such that among any three there exists a pair at the distance less than 1. Prove that there exists a circle of radius 1 that contains at least 13 of the given points
5. Suppose $f(x)$ is a polynomial with integral coefficients and $F(x) = 2$ for three different integers, a, b and c. Prove that for no integer x can $f(x)$ be equal to 3.

   Hint. Prove first that $f(p) - f(q)$ is divisible by $p - q$ for p, q integers. Then use this fact to prove the result.