CS105L: Discrete Structures
I semester, 2006-07

Homework # 4

Due before class on **Friday, August 25, 2006**

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In the following use only the notation described here.

- $\texttt{in}(x, A)$ is True if the element x belongs to the set A.
- $\mathcal{P}$ is the set of all predicates. Every predicate takes some arguments and returns a truth value e.g. the predicate `in` is an element of $\mathcal{P}$ and takes two arguments, the first an element and the second a set. $\texttt{in}(a, B)$ is true if $a \in B$.
- If needed you can also use $\texttt{eq}(x, y)$ which is True if x and y are equal.

Do not use standard mathematical notation like $\in$ etc. Use only logical operators, a colon (:) with quantifiers, the predicates `in` and `eq` and any other predicate whose logical expression can be built from these (you have to show the logical expression for each predicate you use.) Also use uppercase (capital) letters to denote sets and lowercase (small) letters to denote elements of sets.

Write logical expressions for the following:

1. $\texttt{subset}(A, B)$: True if A is a subset of B.
2. $\texttt{function}(f, A, B)$: The relation f is a bijection from A to B.
3. Axiom of Choice: We are given a family of sets $\mathcal{F}$ and an index set I, and a bijection from I to $\mathcal{F}$ i.e. a predicate $\texttt{index}(a, B)$ which is true if $a \in I$ is the index of set $B \in \mathcal{F}$. The axiom says that there is a function f on I which maps $a \in I$ to a $b \in B$ where a is the index of the set B.
4. Bernstein's Theorem: Given two sets A and B, either $|A| \leq |B|$ or $|B| \leq |A|$.
5. $\texttt{PartialOrder}(A, p)$: The relation p is a partial order on the set A.
6. $\texttt{WellOrder}(A, p)$: The relation p well orders the set A.
7. $\texttt{Chain}(A, p, B)$: Given a partially ordered set (A, p), where p is the partial order, a subset B of A forms a chain.
8. Zorn's Lemma: If every chain in a partially ordered set (A, p) has an upper bound in A then A has a maximal element.
9. Well Ordering Principle: For every set A there is a well ordering.