

CS105L: Discrete Structures
I semester, 2006-07

Homework # 3

Due before class on **Friday, August 18, 2006**

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August 10, 2006

1. Consider the following formula, A , of the propositional calculus:

$$p \Rightarrow q \Rightarrow r \vee q \equiv p \wedge q \Rightarrow \neg r \vee q$$

- (a) Using the simple grammar defined for the propositional calculus in class, give three different derivations for A .
 - (b) Do your three different derivations differ in at least one subformula from each other? Take one of your derivations and list all the subformulas for it.
 - (c) For the derivation you chose in the previous part list all the interpretations in which $v(A)$ is true.
 - (d) Now, chose a derivation in which $p \Rightarrow q$ is a subformula. Is $A\{p \Rightarrow q \leftarrow q \Rightarrow p\}$ logically equivalent to A ? If not, give an interpretation in which the two differ. Is $A\{p \Rightarrow q \leftarrow \neg q \Rightarrow \neg p\}$ logically equivalent to A ?
2. Prove the following in the Hilbert system using only the axioms, modus ponens and the deduction rule:
 - (a) $\vdash (A \Rightarrow B) \Rightarrow [(B \Rightarrow C) \Rightarrow (A \Rightarrow C)]$
 - (b) $\vdash [A \Rightarrow (B \Rightarrow C)] \Rightarrow [B \Rightarrow (A \Rightarrow C)]$
 3. Using all the deduction rules (and axioms and MP) prove in the Hilbert system that

$$\vdash (A \Rightarrow \neg A) \Rightarrow \neg A$$