CS105L: Discrete Structures
I semester, 2006-07

Homework # 1

Due before class on **Friday, August 4, 2006**

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1. Find the fallacy in the proof of the following "theorem."

   Theorem 1.1 *A symmetric and transitive binary relation is an equivalence.*

   Proof. Let $\mathcal{R}$ be a symmetric and transitive binary relation on a set A. For any pair of elements $(a, b) \in \mathcal{R}$, it follows from symmetry that $(b, a) \in \mathcal{R}$. Further, from transitivity it follows that if (a, b) and (b, a) are in $\mathcal{R}$ then (a, a) and (b, b) are in $\mathcal{R}$. Hence $\mathcal{R}$ is also reflexive and therefore it is an equivalence. $\square$

2. Can you prove that there exists no bijection between $\mathbb{N}^\omega$ and $\mathbb{N}$?

3. Given any preorder $\mathcal{R}$ on a set A, prove that the *kernel* of the preorder defined as $\mathcal{R} \cap \mathcal{R}^{-1}$ is an equivalence relation.

4. Consider any preorder $\mathcal{R}$ on a set A. We give a construction of another relation as follows. For each $a \in A$, let $[a]_\mathcal{R}$ be the set defined as $\{b \in A | a\mathcal{R}b \text{ and } b\mathcal{R}a\}$. Now consider the set $B = \{[a]_\mathcal{R} | a \in A\}$. Let $\mathcal{S}$ be a relation on B such that for every $a, b \in A$, $[a]_\mathcal{R}\mathcal{S}[b]_\mathcal{R}$ if and only if $a\mathcal{R}b$. Prove that $\mathcal{S}$ is a partial order on the set B.