CS105L: Discrete Structures
I semester, 2005-06

Tutorial Sheet 9: Graph Theory: Matchings

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October 25, 2005

1. Draw two examples of an edge maximal triangle-free graph on ten vertices.

2. Derive Hall's marriage theorem from Tutte's theorem.

3. Let A be a finite set with subsets $A_1, \ldots, A_n$ and let $d_1, \ldots, d_n$ be natural numbers. Show that there are disjoint subsets $D_k \subseteq A_k$, with $|D_k| = d_k$ for all $k \leq n$, if and only if

$$\left| \bigcup_{i \in I} A_i \right| \geq \sum_{i \in I} d_i$$

for all $I \subseteq \{1, \ldots, n\}$.