# CS105L: Discrete Structures
# I semester, 2005-06

Tutorial Sheet 8: Graph Theory continued

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1. An *edge colouring* of a graph is an assignment of colours (or just natural numbers) to the edges of a graph. A *colour class* in a edge coloured graph is a maximal set of edges which have been assigned the same colour. A graph is said to be properly edge coloured if no vertex has more than one edge incident on it from the same colour class. The *edge chromatic number* of a graph, $\chi'(G)$ is the minimum number of colours required to properly edge colour a graph. Prove the following theorem due to Vizing:

   **Theorem 8.1** (**Vizing**) *For a graph $G$ of maximum degree $\Delta$:*

   $$\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1.$$

2. An *independent set* in a graph $G$ is a set of vertices which induce a subgraph on $G$ which has no edges. The *chromatic number* of $G$, denoted $\chi(G)$, is the minimum number of independent sets which cover the entire graph i.e. the minimum number of independent sets whose union is the entire vertex set, $V$, of $G$. If $d$ is the maximum degree of the graph show that

   $$\chi(G) \leq d + 1.$$

3. A graph is said to be an $(\alpha, \beta, n)$-*expander* if it has $n$ vertices and $\alpha$ is the largest number such that every set of size at most $\beta n$ has escape ratio at least $\alpha$. Let us define $G^k$ to be the graph obtained from $G$ by putting edges between all pairs of vertices which have a path of length at most $k$ between them in $G$. Can we say that $G^2$ is an $(\alpha_2, \beta_2, n)$-expander? What are the values of $\alpha_2$ and $\beta_2$? Generalize this to $k$.