CS105L: Discrete Structures
I semester, 2005-06

Tutorial Sheet 6: Recurrences

Instructor: Amitabha Bagchi

September 13, 2005

1. Let a_r denote the number of subsets of $\{1, 2, \ldots, r-1, r\}$ which do not contain two consecutive numbers. Determine a_r.

2. There are two types of particles inside a nuclear reactor. In every second an α particle will split into three β particles and every β particle will split into an α particle and two β particles. If there is a single α particle at time $t = 0$ then how many particles are there in all at time $t = 100$?

3. Solve the following difference equations:

   (a) $a_r^2 - 2a_{r-1}^2 = 1$, given that $a_0 = 2$.

   (b) $a_r^2 - 2a_{r-1} = 0$, given that $a_0 = 4$.

   (c) $a_r = \sqrt{a_{r-1} + \sqrt{a_{r-2} + \sqrt{a_{r-3} + \sqrt{\cdots}}}}$, given that $a_0 = 4$.