CS105L: Discrete Structures
I semester, 2005-06

Tutorial Sheet 5: Generating Functions

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Note: Try to use generating functions to solve all problems, even the ones which do not explicitly mention them. It could also be useful to attempt some of the counting or summation problems both with and without generating function.

1. Let a_r denote the number of ways to seat 10 students in r chairs so that no two students sit in adjacent chairs. Determine the generating function of this numeric function.

2. In how many ways can $3r$ balls be chosen from $2r$ red balls, $2r$ blue ballls and $2r$ green balls?

3. Evaluate the following sums:

   (a)
   $$\binom{n}{1} + 2 \cdot \binom{n}{2} + \cdots + i \cdot \binom{n}{i} + \cdots + n \cdot \binom{n}{n}$$

   (b) Given that $k \leq m$ and $k \leq n$
   $$\binom{n}{0} \cdot \binom{m}{k} + \binom{n}{1} \cdot \binom{m}{k-1} + \binom{n}{2} \cdot \binom{m}{k-2} + \cdots + \binom{n}{k} \cdot \binom{m}{0},$$

   (c)
   $$\binom{2n}{n} + \binom{2n-1}{n-1} + \cdots + \binom{2n-i}{n-i} + \cdots + \binom{n}{0}$$

4. Prove that
   $$\binom{r}{1}^2 + \binom{r}{2}^2 + \cdots + \binom{r}{i}^2 + \cdots + \binom{r}{r}^2 = \binom{2r}{r}$$

   And also that the generating funtion for $a_r = \binom{2r}{r}$ is
   $$\hat{a}(z) = (1 - 4z)^{-1/2}$$