CS105L: Discrete Structures
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Tutorial Sheet 4: Pigeonhole Principle

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1. Consider any five points in the interior of an equilateral triangle of side 1. Show that there are two points which are at most $1/2$ units apart.

2. Consider any five points in the interior of a square of side 1. Show that there are two points which are at most $1/\sqrt{2}$ units apart. Is this this the best possible bound i.e. is there a placement of five points such that the maximum interpoint distance is exactly $1/\sqrt{2}$.

3. Show that among any 9 points inside a triangle of area 1 there are three points which form a triangle of area at most $1/4$.

4. Show that given any 9 points inside a triangle of area 1 there is a triangle of area $1/12$ which does no contain any of those 9 points. Can you do better than $1/12$?

5. Let A be any set of 20 numbers chosen from the arithmetic progression $1, 4, 7, \ldots, 100$. Prove that there must be two distinct integers in A which sum to 104.

6. Suppose $f(x)$ is a polynomial with integral coefficients and $F(x) = 2$ for three different integers, a, b and c. Prove that for no integer x can $f(x)$ be equal to 3.

   Hint. Prove first that $f(p) - f(q)$ is divisible by $p - q$ for p, q integers. Then use this fact to prove the result.