

CS105L: Discrete Structures

I semester, 2005-06

Tutorial Sheet 3: Set Theory

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1. Given Zorn's Lemma prove the Axiom of Choice.

Hint. Given a family of sets \mathcal{C} indexed by an index set I , consider a subset P on the elements of $I \times \cup_{i \in I} X_i$ such that each element e of P is a partial function from I to $\cup_{i \in I} X_i$ with the property that $e(i)$ is either not defined or $e(i) \in X_i$. Define an appropriate partial order on P and then use Zorn's Lemma.

2. Given Zorn's Lemma prove Bernstein's Theorem i.e. Given Zorn's Lemma show that given two sets X and Y there is either a bijection from a subset of X to Y or a bijection from a subset of Y to X .