CS105L: Discrete Structures
I semester, 2005-06

Tutorial Sheet 12: Discrete Probability

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1. Let (Ω, P) be a finite probability space in which each outcome (also known as elementary event) has the same probability. Show that if $|\Omega|$ is a prime number then no two non-trivial events (i.e. events which are not $\emptyset$ or Ω) can be independent.

2. Assume that the probability of a girl or boy being born are equal.

   (a) For a certain family with two children we know that at least one of them is a boy. What is the probability that both the children are boys?

   (b) It is found that a certain family with two children has a son. What is the probability that this boy's sibling is a girl?

3. For a given positive integer n we know that there is a family that can have 1, 2,$\ldots, n$ children with probability $p_1, p_2, \ldots p_n$ such that $\sum_{i=1}^{n} p_i = 1$. Also for any family with i children each of the 2^i probability distributions over boys and girls are equally likely. If a family has no girls, what is the probability that it has only one child?

4. We select two points uniformly independently from a line segment of unit length. The two points partition the segment into three sub-segments.

   (a) What is the expected length of the smallest segment?

   (b) What is the probability that the three sub-segments can form a triangle?

5. A bin contains m white and n black balls. We take out the balls uniformly at random from the bin without replacing them. What is the expected number of black balls left when all the white balls have been taken out?

6. We have n balls numbered 1 to n in a bag. We select a ball at random from the bag and note it's number. If this number, i, is 1 we stop otherwise we continue to remove balls from the bag uniformly at random (with replacement) till we get a ball which is numbered less than i. What is the expected number of balls we need to pick till we stop?