CS105L: Discrete Structures
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Tutorial Sheet 10: Graph Theory: Connectivity

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For the first three exercises, let G be a graph and $a, b \in V(G)$. Suppose that $X \subseteq V(G) \setminus \{a, b\}$ separates a from b in G. We say that X separates a from b *minimally* if no proper subset of X separates a from b in G.

1. Show that X separates a from b minimally if and only if every vertex in X has a neighbour in the component C_a of $G \setminus X$ containing a, and another in the component C_b of $G \setminus X$ containing b.

2. Let $X' \subseteq V(G) \setminus \{a, b\}$ be another set separating a from b, and define C'_a and C'_b acoordingly. Show that both

$$Y_a = (X \cap C'_a) \cup (X \cap X') \cup (X' \cap C_a)$$

   and

$$Y_b = (X \cap C'_b) \cup (X \cap X') \cup (X' \cap C_b)$$

   seperate a from b.

3. Do Y_a and Y_b separate a from b minimally if X and X' do? Are $|Y_a|$ and $|Y_b|$ minimum for vertex sets separating a from b if $|X|$ and $|X'|$ are?

4. Let X and X' be minimal separating vertex sets in G such that X meets at least two components of $G \setminus X'$. Show that X' meets all the components of $G \setminus X$, and that X meets all the components of $G \setminus X'$.