CS105L: Discrete Structures
I semester, 2005-06

Homework # 5

Due before class on **Thursday, September 22, 2005**

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September 15, 2005

1. Write the following graph properties as logical statements. Use the predicate $\texttt{in}(x, Y)$ to denote set membership, $\texttt{subset}(A, B)$ and also $\texttt{eq}(a, b)$ which returns T if a and b are the same. Don't use this to compare numbers. For numbers, use only tt isOne(n) which returns T if n is 1. Use the symbol V to denote the vertex set of a graph, and E to denote the edge set of the graph. Also, use the predicate $\texttt{adjacent}(u, v)$ which applies to vertice u and v and returns T if there is an edge between u and v.

   (a) Minimum degree of the graph is 2.

   (b) The graph is *not* a complete graph.

   (c) The graph is triangle-free (i.e. there is no 3-cycle in the graph.)

   (d) The graph is a star (i.e. one vertex of degree $n-1$ connected to $n-1$ vertices of degree 1.)

   (e) There is a path of length k between the vertices u and v.

   (f) The graph is bipartite.

2. Count the following:

   (a) The number of graphs on n vertices.

   (b) The number of paths of length k between two vertices u and v in a complete graph on n vertices.

   (c) The number of bipartite graphs on n vertices.

3. For some natural number d, let's say the vertex set of a graph is labelled with the strings from $\{0,1\}^d$ i.e. each vertex has a unique label which is a d-bit string and every d-bit string corresponds to a vertex. Further we say that there's an edge between two vertices if their labels differ in exactly one position. This graph is known as the *d-dimensional cube.* Determine the average degree, number of edges, diameter, girth and circumference of this graph. Give proofs of all your claims.